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A THEORY OF TURBULENCE IN A HOMOGENEOUS FLUID INDUCED BY AN OSC--ETC(U)

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Departments of Earth & Planetary Science
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Baltimore, Maryland

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By
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Technical Report No. 10 (Series C)

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A THEORY OF TURBULENCE IN A HOMOGENEOUS FLUID INDUCED BY AN OSCILLATING GRID

By

Robert R. Long

The Johns Hopkins University, Baltimore, Maryland 21218

Several investigators have run experiments to study turbulence in a container of homogeneous fluid caused by a grid extending horizontally across the box and oscillating vertically. The more recent experiments indicate that the root-mean-square horizontal velocity σ_u is proportional to the oscillation frequency f and is inversely proportional to distance from the grid, and that the integral length scale ℓ is proportional to this distance and independent of f. A theoretical model is based on the Navier-Stokes equations and the no-slip boundary condition but contains no other assumptions. The statistically steady behavior is as observed in the experiment. In the unsteady case, the front separating turbulent and non-turbulent fluid propagates at a speed inversely proportional to the square root of time.

The problem has importance with respect to the mixed layer in the upper ocean or the lower atmosphere. The present model does not consider buoyancy effects but if a passive contaminant is present the mean concentration is governed by the one-dimensional diffusion equation with constant turbulent coefficient of diffusion.

Bouvard and Dumas (1967) constructed an experiment with configuration represented in figure 1. A grid is oscillated vertically with stroke S and frequency f in a container of homogeneous fluid. The motion induced is turbulent and interest is attached to the root-mean-square horizontal velocity σ_u and the integral length scale ℓ as functions of distance from the grid (or of distance z from some virtual energy source), the frequency f, the stroke S and the lengths $M_1, M_2, \ldots$ characteristic of the grid geometry and position. The problem was also investigated experimentally by Thompson and Turner (1975) and very recently by Hopfinger and Toly (1976). Thompson and Turner's data suggest that $\sigma_u \ell$ is independent of distance from the grid and that σ_u is proportional to the frequency of oscillation of the grid f at any given level. These two behaviors were also found by Hopfinger and Toly. They also measured ℓ and found $\ell \propto z$, so that $\sigma_u \propto z^{-1}$.

To obtain an understanding of this experiment, we solve the problem of statistically steady, finite motions in a Newtonian fluid filling infinite space induced by a steady disturbance source on the plane $z = 0$ whose properties are independent of x and y. The properties of the motion should be the same as those in the experiment if the frequency of oscillation of the grid is large and the stroke and its geometrical lengths are small. A complete comparison, however, requires relations between some parameter or parameters characterizing the theoretical source and $f, S, \nu, M_1, M_2, \ldots$.

We begin by considering a thin "source" layer $-d \leq z \leq d$. The layer may contain a grid or some other arrangement conveniently constructed in the laboratory or in the mind. Integrating the averaged energy equation from d to z, we get

$$0 = -F^3 + E^3 - D^3 \tag{1}$$

where F^3 and E^3 are the energy fluxes $w(p/\rho + q^2/2)$ averaged over the planes at distances z and d respectively from the plane $z = 0$, and D^3 is the total energy dissipation,

$$D^2 = -\nu \int_d^z \overline{(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w)}\,dz \qquad (2)$$

In the expressions above, u, v, w are the velocities in the x, y, z directions, p is pressure, ρ is the uniform density and q is the fluid speed.

If d is finite and non-zero, E^2 is finite and non-zero if we require that the velocities at all z be finite and non-zero. Then, for example, σ_u will vary in some complicated way with E^2, d, z, ν and other quantities that depend on the nature of the energy source. Evidently we may improve the situation by letting $d \to 0$. Let us assume, tentatively, that we have the freedom to construct an energy source that produces disturbances near $z = d$ with the following properties:

(a) The only length scale is d, i.e. derivatives with respect to x, y, z vary as d^{-1} when operating on any non-constant quantity.

(b) The velocity varies as d^{-n}, where n is a universal constant.

(c) The vorticity ζ is proportional to the velocity scale divided by the length scale, i.e., d^{-n-1}.

(d) The average energy flux on the plane $z = d$ is of order d^{-3n}.

We will see later that we have this freedom.

Since F^2 is finite and non-zero, D^2 cannot be zero as $d \to 0$. If D^2 is finite and non-zero, E^2 is finite and non-zero. But E^2/d is proportional to the dissipation function ϵ just outside the source layer. Using properties (a)-(d), we get

$$\epsilon \propto \zeta^2 \propto d^{-2n-2} \propto d^{-1}$$

This means that the velocities at $z = d$ tend to zero so that E^2 is zero. The contradiction reveals that E^2 and D^2 are both large and since F^2 is finite $E^2 \sim D^2$.

Again, E^2/d is proportional to the dissipation function just outside the source layer so that

$$d^{-3n-1} \sim d^{-2n-2}$$

or $n = 1$. Thus E^2 tends to infinity but E^2d^3 tends to a finite quantity, which we call K^3. The constant K and the viscosity ν are the only two parameters of the problems. Dimensional analysis yields, for example

$$\frac{\sigma_u z}{K} = A_1 \quad , \quad \frac{\ell}{z} = A_2 \tag{3}$$

where A_1 and A_2 are functions of the Reynolds number K/ν. As $K/\nu \to \infty$, they become universal constants.

We now demonstrate that a flow near $z = d$ may be chosen with the four properties listed above. We put doublets of strength κd^2 (finite κ) at all points $(nd, md, 0)$ where n and m are all of the positive and negative integers. Doublets with axes in the direction of x positive and x negative correspond to $n+m$ even and $n+m$ odd, respectively. We also impose a rigid plane at $z = 0$ with infinitesimal holes for the doublets. The no-slip condition insures that there will be vorticity. The length scale of the motion near $z = d$ is d times a function of κ/ν and so has the property (a). The velocity scale near $z = d$ is κ/d and so has the property (b). There is vorticity and the scale is proportional to d^{-2} so has the property (c). Finally, an irrotational fluid in steady state has a uniform value of $p/\rho + q^2/2$ and it is then clear that the correlation in the mean energy flux term is zero. In the present problem there is loss of energy because there is vorticity and viscosity so that on the average $(p/\rho + q^2/2)$ will be less for incoming fluid than for outgoing fluid at the plane $z = d$. The difference is κ^2/d^2 times a function of κ/ν and the mean flux on $z = d$ is of order κ^3/d^3, satisfying property (d).

The variation of σ_u and ℓ with z in (3) agrees well with experiment. Equation (3) also reveals that σ_u is proportional to K. If we now neglect the effect of viscosity, the frequency f is the only experimental parameter involving time so that σ_u should be proportional to f as observed. Notice, however, that a relationship between K and f is analogous to the drag coefficient in shearing flow and the latter is known to involve the Reynolds number, although weakly, when the Reynolds number is large (Monin & Yaglom, 1971). Therefore, σ_u/f may, on close inspection, vary slowly with f.

The experiments and theory relate to the important problem of the mixed layer in the upper ocean and the lower atmosphere, where, however, buoyancy is important dynamically. The mean buoyancy $\bar{b}$, or more generally, the concentration of any contaminant is subject to the equation

$$\frac{\partial \bar{b}}{\partial t} = \frac{\partial}{\partial z}\left(K_D \frac{\partial \bar{b}}{\partial z} \right) \tag{4}$$

where K_D is the eddy diffusivity. If the buoyancy variation is weak or, more generally, if the contaminant is dynamically unimportant, K_D is proportional to the constant eddy viscosity and (4) may be solved for $\bar{b}$ subject to initial and boundary conditions.

Finally, consider the initial value problem when the energy source begins at $t = 0$ in an infinite fluid at rest. A front separating turbulent and non-turbulent fluid will move away at a speed $u_e = \varphi(K, \nu, t)$. Therefore,

$$\frac{u_e t^{\frac{1}{2}}}{K^{\frac{1}{2}}} = A_3 \tag{5}$$

where A_3 is a function of K/ν.

Acknowledgement.

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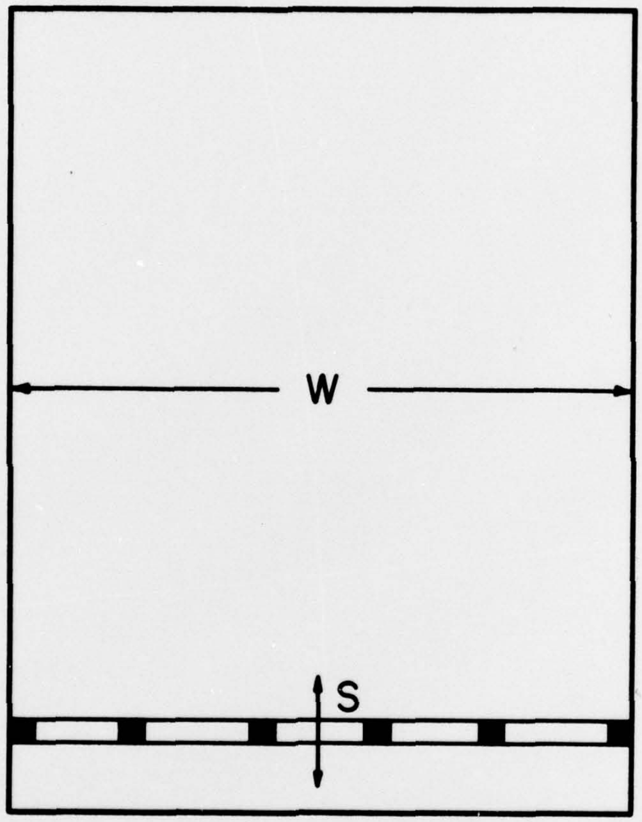
Figure 1. Schematic diagram of the experimental apparatus.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